THE APPROXIMATE NORMAL TEMPERATURE AS A FUNCTION OF THE LATITUDE, ELEVATION, TIME OF DAY, AND DAY OF THE YEAR.

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SYNOPSIS.

In an earlier contribution, it was shown that the following empirical equation gives the normal temperature at any hour of any day with an average error of 2.75° F.:

$$t = M + \frac{R}{2}\cos d + \frac{r}{2}\cos h + \frac{V}{4}\cos d \cos h \qquad \ldots (1)$$

in which M is the average annual temperature, R the annual range or the difference between the mean of the day with the highest normal temperature and that with lowest, r the daily range, V the difference between the daily ranges at the coldest and warmest times of year, d the time of year, and h the time of day. Beginning at the time of maximum temperature, d and h are expressed in degrees 0 to 360, and with 180 at the time of minimum temperature even though this makes each degree of h from 180 to 360 represent shorter intervals of time than does each from 0 to 180.

In this new paper it is shown that the constants in the above equation can be expressed fairly accurately by the following equations which apply to the United States east of the 100th meridian:

$$M = 110 - 1.4l - 0.002\,E \qquad \ldots (2)$$

where M is the mean annual temperature, l the latitude, and E the elevation (expressed in feet) above sea level;

$$R = -24 + 1.8l \qquad \ldots (3)$$

where R stands for the annual range in temperature and l for the latitude. The daily range r is sensibly constant and has a value of 18° F., and V is zero.

Equation (1) with these substitutions reduces to

$$t = 110 - 1.4l - 0.002\,E + (0.9l - 12)\cos d + 9\cos h \quad \ldots (4)$$

For the arid West the following equation applies very approximately:

$$M = 121 - 1.4l - 0.0033\,E \qquad \ldots (5)$$

The other values are sensibly constant all over this area and have the following values: One-half the annual range is 23°, one-half the mean daily range is 11°, and one-fourth the variation in the daily range 3°.

Equation (1) becomes on substitution of these values for the arid West of the United States:

$$t = 121 - 1.4l - 0.0033\,E + 23\cos d + 11\cos h + 3\cos d\cos h \quad . \ (6)$$

The mean error in the use of equations (2), (3), and (5) for places all over the United States except the humid Pacific coast was ±1.1° F., and in the use of equations (4) and (6) giving the normal temperature as a function of the latitude, elevation, and time ±3° F.

In an earlier contribution,[1] the normal temperature at any hour of the day was expressed as a function of the annual mean, the annual range, the daily range, the time of year, and the time of day. It is the purpose of this paper to consider this relationship further, introducing the additional considerations of latitude and elevation. Only two of the three sections into which the United States was divided will be considered here, namely, the region east of the 100th meridian, and the region between the Rocky Mountains and the Sierra Navada-Cascade Mountains. To facilitate consideration of the new factors enumerated above, places of nearly the same latitude were selected in considering the effect of elevations, and, in considering latitude, places of nearly the same elevation were selected. Where these conditions were nearly, but not quite satisfied, a slight correction was introduced to bring the data to a common latitude or elevation, respectively.

Mean annual temperature and elevation above sea level.—Figure 1 represents the elevation above sea level and the mean annual temperature of about 100 cities distributed over the eastern part of the United States, these values having been reduced to the same latitude. The arid West is shown by the dotted line. A range from sea level to 2,000 feet was selected in the first case and from 3,000 to 6,000 feet above sea level in the second because most of the territory of these divisions has these respective ranges of elevation. The temperature gradient is not constant, the value for the low section being 0.002 and for the high western section 0.0033 (500 and 300 foot rise to 1° F. colder); and yet over the range of each separate land division the gradient is sufficiently constant for all practical purposes, as shown from the fact that the straight line fits the data so well.

Temperature and latitude.—Figure 2 shows the manner in which the mean annual temperature M changes with the latitude l over the East.[2] All temperatures were reduced to the equivalent sea-level values. The arid West is shown by the dotted line. For both sections it is clearly seen that the temperature decreases rather regularly as the latitude increases, the change being 1.4° F. for each degree of latitude, the gradient being the same for both sections.

Temperature, latitude, and elevation.—The equation which fits the above data for the eastern division is:

$$M = 110 - 1.4l - 0.002\,E \qquad \ldots (2)$$

where M represents the mean annual temperature, l the latitude, and E the elevation above sea level in feet. This equation checked against actual values for 26 cities selected at random over 23 States showed a departure of ±0.9° F. That for the arid West is:

$$M = 121 - 1.4l - 0.0033\,E \qquad \ldots (5)$$

This equation checked against cities widely separated over all the arid States of the Great Basin gave a departure of ±1.3° F.

Range of the annual march.—Figure 5 shows the manner in which the difference in temperature between summer and winter, or the annual range in temperature, varies with the latitude for the eastern part of the United States. In general, the range is least for oceanic sections and most for continental areas, but probably because of the prevailing westerly winds the Atlantic Ocean has slight effect on the coast cities, the range being nearly the same there as for cities of the same latitude in the interior. The Gulf of Mexico seems to have a marked effect on the annual variation. It has a minimum value on the shores of the Gulf and increases very regularly by 1.8° F. for each degree of latitude north. The equation representing this relationship for the eastern division is:

$$R = -24 + 1.8l \qquad \ldots (3)$$

where R stands for the annual variation or range and l for the latitude. Results from equation (3) compared with

[1] West, Frank L. A Simple Equation of General Application for the Normal Temperature in Terms of the Time of Day and the Day of the Year. Mo. WEATHER REV., July, 1920, 48: 394-396. Cf. also *Science*, Dec. 24, 1920, p. 611.

[2] The greater the annual rainfall the smaller the mean annual temperature, the correction being approximately 0.02° F. for each inch of rain. The area considered has a rainfall varying from 35 to 50 inches, and this variation of 15 inches would only amount to less than ½° F.; hence, the rainfall term was not used in the final equation.

actual values of 26 cities representing 23 States showed a mean departure of $\pm 0.9°$ F.

In the arid West, which is shut in by the Rocky and Sierra Nevada Mountains, the average annual range for 25 cities scattered over the southern half of the division differed from the average of a like number for the northern half of the same area by only $\frac{1}{4}°$ F., showing that the range is not simply a function of the latitude for this territory. The mean annual range for the entire district was 46° F. One-half of this value, or 23, is used in equation (6). The mean departure from this value of 23° by the different cities is 1.0° F.

Daily range in temperature.—The difference between the 6 a. m. and 3 p. m. temperatures (maximum and minimum) is nearly constant for all points of the eastern division and for all seasons of the year and has a value of approximately 18° F. One-half of this value, or 9°, is

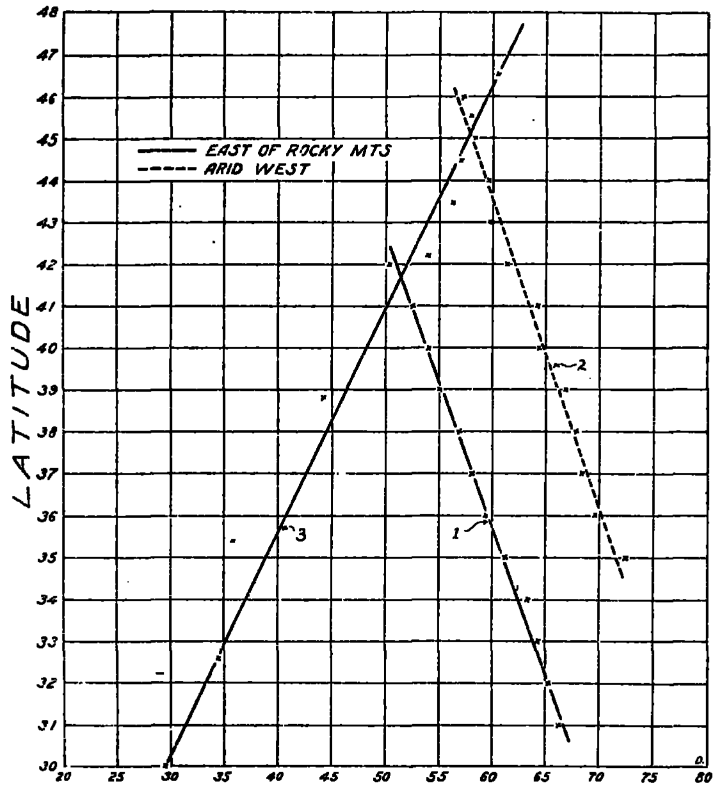

FIG. 1.—Graphs showing the changes in the mean annual temperature (in degrees Fahrenheit) with latitude, in the eastern and arid West portions (1) of the United States. Graph showing how the annual range in temperature (in degrees Fahrenheit) changes with the latitude for the United States east of the Rocky Mountains.

used in the formula, and the departure from this mean of the different cities was 1.4° F.

For the arid West the mean daily range, r, for all the days of the year is 22°, and the difference, V, between this value for a day in summer and for a day in winter is 12°. One-half and one-fourth of these values, 11 and 3, respectively, are used in equation (6) and the values for individual cities do not differ appreciably from these values.

The coefficient of the last term of equation (4) for the eastern division and the last three coefficients of equation (6) for the entire western or arid region were taken as constants. The actual variations in these values for different cities are usually no more than 1° F. It should be borne in mind that these constants simply specify the height of the crests of the curves above the mean. They are half the annual range and half the daily range. Even if the value for a particular city were to depart as much as 2° F. from these, yet the actual curve of normals for the city when superposed on the curve of means for the area would simply have its crest 1° F. above and its trough 1° F. below that for the equations, and the curves would coincide at the mean and very nearly so all the way up the curve until very nearly to the crest. There, of course, the error would be the full 1° F. The small variations, therefore, in these constants make less difference in the final result than would at first appear and, besides, the variations in the constants themselves are small.

When these values for M, R, r, and V are substituted in the general equation given in the earlier paper, viz,

$$t = M + \frac{R}{2}\cos d + \frac{r}{2}\cos h + \frac{V}{4}\cos d \cos h$$

the approximate normal hourly temperature for a city in the eastern United States or for one in the arid West can

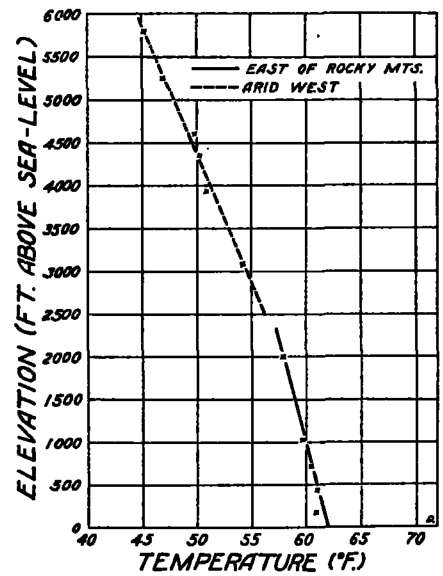

FIG. 2.—Graph showing the changes in the mean annual temperature with elevation in the United States.

be computed from the latitude and altitude of the place, the equations becoming

$$t = 110 - 1.4l - 0.002\,E + (0.9l - 12)\cos d + 9\cos h \quad \text{(East)} \qquad (4)$$

$$t = 121 - 1.4l - 0.0033E + 23\cos d + 11\cos h + 3\cos d\cos h \quad \text{(arid West)} \qquad (6)$$

These final equations (4) and (6) are interesting, inasmuch as they apply to such a large area of country, and they are thus rather general. However, their test is: Do they fit the facts? The mean difference between the actual normal hourly temperatures and those obtained by equation (1) was 2.75° F. In using equations (2), (3), and (5) it was 1.1° F., and with the use of equations (4) and (6) it was $3\frac{1}{2}°$ F. Actual temperatures differ from the normals, and equations (4) and (6) will give the actual temperature experienced at a particular hour on a particular day usually within 5° F.

Equations (4) and (6) have practical value in such cases as the determination of early morning temperatures where heating to protect crops from frost is practiced, in calculating hourly values where thermograph records have not been taken, and for engineers engaged in laying concrete in determining the normal time in the spring and fall when freezing temperatures are experienced during working hours.